TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 680

APPROXIMATE CALCULATION OF MULTISPAR CANTILEVER AND SEMICANTILEVER WINGS WITH PARALLEL RIBS UNDER DIRECT AND INDIRECT LOADING

By Eugen Sänger

Zeitschrift für Flugtechnik und Motorluftschiffahrt
Vol. 23, No. 9, May 14, 1932
Verlag von R. Oldenbourg, München und Berlin

Washington
August, 1932

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APPROXIMATE CALCULATION OF MULTISPAR CANTILEVER AND SEMICANTILEVER WINGS WITH PARALLEL RIBS UNDER DIRECT AND INDIRECT LOADING*

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The calculation of parallel-ribbed multispar wing structures, which are statically indeterminate to a high degree, presents quite serious difficulties, even with the use of partial differential equations. This statement is especially true when there is a large number of spars, variable spacing of the spars, different moments of inertia of the individual spars, variable moments of inertia of the ribs, closely spaced ribs, polygonal arrangement of the spars in plan, or combinations of these cases.

In such cases the calculation is greatly simplified by the customary assumption of rigid ribs, without appreciably affecting its accuracy, if the magnitude and direction of the consequent errors are estimated and allowed for in the final result.

For illustrating this method of calculation, a wing structure of the following geometrical characteristics is assumed: Spars of any number n and of any inertia moment, for which it is assumed that the cross sections of different spars, located at equal distances from the airplane axis, have inertia moments reduced in the same proportion as compared with the inertia moment at the wing root, a requirement which accords with the general condition that the spars must have the same bending resistance. Furthermore, it is assumed that the spars are inclined to-

*"Zur genäherten Berechnung vielholmig-parallelstegiger, ganz- und halbfreitragender, mittelbar und unmittelbar belasteter Flügelgerippe." Zeitschrift für Flugtechnik und Motorluftschiffahrt, May 14, 1932, pp. 245-250.
This article is supplementary to a previous article by the same author published in Z.F.M., Oct. 28, 1931, pp. 597-603, a translation of which was issued as T.M. No. 662, N.A.C.A., 1932.

ward the longitudinal axis of the airplane by any desired angle, possibly varying from spar to spar, with the limitation that the extended spar axes must all intersect at a single point, which may be infinitely distant, and that, of course, both wings must be symmetrical with respect to the axis of the airplane. Lastly we assume any desired number of ribs $2m + 1$ at any desired distance from one another and parallel to the longitudinal axis of the airplane.

Figure 1 shows the plan of this very common type of wing structure, whose spars and ribs may be either solid or lattice beams. It is of interest to note that, because of the assumption of no torsional rigidity of the individual spars, the center rib cannot generally be omitted when the angle of the spars to the airplane axis is other than $\pi/2$. If the center rib is omitted, then the spars must be perpendicular to the airplane axis, at least between the ribs $+1$ and -1.

The structural investigation itself proceeds essentially according to the following line of reasoning. The aerodynamic forces are transmitted from the wing covering to the points of intersection (joints) of the spars and ribs in the form of individual loads, so that, in the loading case A, every joint is subjected to a definite external individual load. In the transition to another loading case, the mutual quantitative relationships of these loads vary only in the direction of the ribs and not in the direction of the spars. The latter circumstance raises the question of letting a certain unit series of loads on the joints of a spar (which is similar to the series of spar loads in the loading cases and into which series it may be converted through multiplication by a constant factor) travel from spar to spar, while all the other spars are not loaded, and of investigating, for each position of the "unit-loading series" on a spar, the effect of this loading on joint forces, spar-joint moments, rib-joint moments, joint displacements, etc. If, in this manner, the effects of all the positions of the individual load series on the question to be investigated are ascertained, the effect of any flight loading case can be determined very simply by the suitable combination and multiplication of the unit load series, especially when the effects of the unit loads are expressed in the form of influence lines.

In the wing structure under investigation the joint displacements $z_{o1} = 0$ and $z_{on} = 0$ are assumed as adequate support conditions. In the calculation of one of the thus-developed load conditions (sometimes one spar under the influence load, all the other spars being without load), the process is as follows.

The influence loads are transferred to the elastic axis of the wing structure, i.e., the connecting line of that cross-sectional point of the wing structure, by the vertical loading of which the cross section suffers displacement but not twisting. All the joint forces $K_{xy}^{(1)}$ are determined for this position. For carrying out the assumed displacement of the influence loads, we have imagined a torsional moment of "unit load times the displacement distance" in every rib.

Our next task is the determination of the position of the elastic axis of the wing structure at the ribs. Any one of the rigid ribs is selected and regarded according to Figure 2 as a rigid beam on n elastic yielding supports for which is sought the point of application of a single load producing a parallel displacement but no twisting of the beam.

If z is the parallel displacement (downward) of the whole beam under the load P_x, then the joint forces are

$$K_1 = k_1\ z; \quad K_2 = k_2\ z; \ldots\ K_{n-1} = k_{n-1}\ z; \quad K_n = k_n\ z$$

in which $k_1, k_2 \ldots k_{n-1}, k_n$ are the support forces which cause a support depression of unity. These n equations suffice, together with the equilibrium conditions against displacement and torsion, for calculating all the unknown quantities. This yields for ζ, in which alone we are interested,

$$\zeta = \frac{a_2\ k_2 + a_3\ k_3 + \ldots + a_{n-1}\ k_{n-1} + a_n\ k_n}{k_1 + k_2 + \ldots + k_{n-1} + k_n}$$

The n forces supporting the rigid rib, whose distance from the longitudinal axis of the airplane is x, must be regarded as produced by cantilever beams of different spans and of different inertia moments variable along their spans, on whose free ends the rib is supported. Since, however, the course of the inertia moment is assumed to be of such a nature that, for like x, all spar inertia moments are like fractions of the original inertia moment and, since we are not here concerned about the absolute values, but only about the mutual relations of the supporting forces K, we can, notwithstanding the correctness of these considerations for the spar inertia moment, introduce the value J_{oy} at the spar root into the subsequent calculations. The individual supporting forces for a unit depression then become:

$$k_1 = 3\ E\ J_{o1}\ \sin^3 \alpha_1/x^3;\ k_2 = 3\ E\ J_{o2}\ \sin^3 \alpha_2/x^3 \ldots k_{n-1} =$$

$$= 3\ E\ J_{o,n-1}\ \sin^3 \alpha_{n-1}/x^3;\ k_n = 3\ E\ J_{o,n}\ \sin^3 \alpha_n/x^3,$$

so that the location of the elastic axis of the first spar follows with

$$\zeta = \sum_{y=o}^{n} a_y\ J_{oy}\ \sin^3 \alpha_y \Big/ \sum_{y=o}^{n} J_{oy}\ \sin^3 \alpha_y \qquad (1)$$

The elastic axis does not therefore generally pass through the c.g. of the actual wing section, nor through the c.p. of the wing covering, but through the contemplated point of intersection of the wing spar.

Our next task, the determination of $K_{xy}^{(1)}$, can now be undertaken in a simple manner. Under the individual loads transferred to the elastic axis, the wing bends without twisting. All the spars must therefore bend equally, whereby, on account of the symmetry of the total loading with respect to the longitudinal axis of the airplane and on account of the rigid middle rib, they work as though they were rigidly held at their roots. The resistance of each spar to the bending force is proportional to its relative stiffness $E\ J_{o,y}\ \sin^3 \alpha_y/x^3$, so that the reactions of the single load P in the individual rib joints can be calculated according to the proportion

$$K_{x,1}^{(1)}:K_{x,2}^{(1)}:\ldots:K_{x,n-1}^{(1)}:K_{x,n}^{(1)}=J_{o,1}\ \sin^3 \alpha_1:J_{o,2}\ \sin^3 \alpha_2:\ \ldots$$

$$\ldots:J_{o,n-1}\ \sin^3 \alpha_{n-1}:J_{o,n}\ \sin^3 \alpha_n \qquad (2)$$

whereby

$$K_{x,1}^{(1)} + K_{x,2}^{(1)} + \ldots + K_{x,n-1}^{(1)} + K_{x,n}^{(1)} = P_x$$

Thus all the attacking forces are known for every individual spar and rib and the stresses can be determined in the usual elementary way.

The joint forces $K_{x,y}^{(2)}$ are produced by the torsional moment then prevailing in every rib, when the external load is transferred to the elastic axis from its actual point of application. This moment causes a twisting of the rib about the static axis. Thereby all the spars must bend by amounts proportional to their distance from the static axis and thus offer resistances to this bending, which are proportional, on the one hand, to the deflection in the elastic region of stress and, on the other hand, to the relative bending rigidity $E\,J_{o,y}\,\sin^3\alpha_y/x^3$. Altogether these resistances offset the torsional moment of the ribs. Figure 3 shows the torsional moment about the static axis of the wing.

For calculating the individual joint pressures $K_{x,y}^{(2)}$, we can therefore use the proportion

$$K_{x,1}^{(2)}:K_{x,2}^{(2)}:\ldots:K_{x,n-1}^{(2)}:K_{x,n}^{(2)} =$$

$$= \zeta\,J_{o,1}\,\sin^3\alpha_1:(\zeta-a_2)\,J_{o2}\,\sin^3\alpha_2:\ldots$$

$$\ldots:(\zeta-a_{n-1})\,J_{o,n-1}\,\sin^3\alpha_{n-1}:(\zeta-a_n)\,J_{o,n}\,\sin^3\alpha_n \quad . \quad .(3)$$

Along with the condition that the sum of all the moments about any given point must equal the external moment, we again have n equations with n unknowns, while all the impressed forces are known for every individual spar and rib. In the calculation of $K_{x,y}^{(1)}$ it has been tacitly assumed that the ribs act independently of one another and that therefore the loading of one rib has no effect on the stresses in another rib. That this is actually the case is evinced by the following consideration.

If a rib is subjected to a single load in the elastic axis or to a torsional moment, reactions are produced in the joints of this rib by the spars, the rib suffers bending stresses (which are assumed to cause no deformation) and is, as a result of the deformations of the spars, displaced or rotated. These deformations take place so that spar cross sections at the same distance from the airplane axis do not change their relative position. If they were in a straight line before the loading, this is still the case after the deformation. The course of the spar inertia moments was chosen in this way. If such a series of spar sectional points happen to be connected by a rib, this rib is either depressed or distorted with the spars, but undergoes no stress itself and so has no effect on the other deformations. If another rib is now loaded in any way, this rib will be stressed and will impart an additional deformation to the whole wing and additional stresses to the spars, the magnitude of the latter stresses being calculated as if the first load were absent. The first-loaded rib will thereby experience no change in stress and none of the other ribs will be stressed at all. We can therefore in fact (assuming perfectly rigid ribs and external loads only in the joints) load each by itself and independently of the other ribs and then simply superpose the effects of all the loads.

By superposing the two load conditions involved in the calculation, we obtain

$$K_{x,y} = K_{x,y}^{(1)} + K_{x,y}^{(2)}$$

as the actual joint forces under the external loads and from this all spar-joint moments L_{xy}, rib-joint moments M_{xy} and joint displacements z_{xy}. All the considerations that led to these results were exact, up to the actual assumption of an infinitely great rib inertia moment. Hence the accuracy of our calculation is determined simply by the effect of this assumption on the result. It is important therefore to determine the magnitude and limits of this influence not only to ascertain the probable error of the method, but also (since its direction is known) to offset it as much as possible. The hitherto disregarded rib deformations must be taken as the starting point of our deliberations in this connection. As the first approximation, these deformations can be determined for every load condition from the inertia moments and the rib stresses M_{xy}. It is not possible to determine by simple means what relation the thus-found flexural curves of the ribs bear to the previously assumed undistorted axis of the ribs, since the mutual independence of the ribs in their finite rigidity no longer exists. According to Figure 4, however, it may be claimed with approximate accuracy that the following relations must exist between the small joint displacements $z_{xy}^{(3)}$ resulting from the transition of the inertia moment of the ribs from an infinite to a finite value under a constant full load P_x, since the sum of the forces and moments cannot change in the assumed deformation.

$$z^{(3)}_{x,1} J_{01} \sin^3 \alpha_1 + z^{(3)}_{x,2} J_{0,2} \sin^3 \alpha_2 + \ldots$$

$$\ldots + z^{(3)}_{x,n-1} J_{0,n-1} \sin^3 \alpha_{n-1} + z^{(3)}_{x,n} J_{0,n} \sin^3 \alpha_n = 0$$

$$a_2 z^{(3)}_{x,2} J_{0,2} \sin^3 \alpha_2 + \ldots + a_{n-1} z^{(3)}_{x,n-1} J_{0,n-1} \sin^3 \alpha_{n-1} +$$

$$+ a_n z^{(3)}_{x,n} J_{0,n} \sin^3 \alpha_n = 0,$$

whereby $z^{(3)}_{xy}$ of the undistorted rib axis may be either positive or negative.

The equations show that the new support pressures for the whole rib yield neither a resultant force nor a resultant moment. These equations clearly indicate the relations between the deformed and undeformed elastic line. If all the flexural curves of the ribs were congruent the thus-found $z^{(3)}_{xy}$ could be used directly for the further calculation. However, if such is not the case, the individual $z^{(3)}_{xy}$ of the different ribs and of the same spar will bear ratios to one another corresponding to the support of the whole wing and the locally variable rigidity of the spars. This mathematically difficult equalization can be made with sufficient accuracy by graphic interpolation, whereby we need to consider only the apparent form of the elastic line of the spar and ribs. The limits, within which the arbitrary interpolation can be made, are very narrow for the corresponding scale.

With the now definitely determined equalized additional joint displacements $z^{(3)}_{xy}$ we can apply the corrections to the joint displacements z_{xy} according to the approximation method and can also improve the joint forces, whereby special attention must be given the sign of the additional pressures $K^{(3)}_{xy}$. With the thus newly obtained and improved joint forces, the rib moments and possibly also the deflections can be improved. Such refinements, however, will generally be dispensed with, since the already attained accuracy far surpasses the regularity of the material characteristics, so that a new uncertainty factor is introduced into the calculation by the graphic interpolation. Hence we have a very simple, accurate and generally applicable method for the calculation of multi-spar wings with parallel ribs, which, in part, constitutes an extension of the customary method of calculating such wings, which, as already remarked, is also based on the assumption of rigid ribs. The extensions consist essentially in

1. Knowledge of the twisting of the wing about the elastic axis, whereby, on the one hand, the method is applicable to the calculation of multispar wings without torsionally rigid covering and, on the other hand, in the case of such wings with torsionally rigid covering, this method makes it possible to estimate to what degree the rigidity of the wing covering is actually utilized for the absorption of the torsional moments.

2. Indication of the limits within which the errors due to the approximate assumption of rigid ribs can be neutralized.

3. Employment of influence lines with the advantages already discussed in Z.F.M., October 28, 1931 (T.M. No. 662, N.A.C.A., 1932) and to receive further consideration here.

This method will now be developed for a few special cases outside the scope of the previously assumed wing and load characteristics.

SEMICANTILEVER WING

In return for the structural and especially for the aerodynamic disadvantages of semicantilever wings, a certain advantage accrues from the fact that the high fuselage enables the bracing of outlying spar joints against the fuselage and a corresponding reduction in the spar-root moments. The most important practical case is the one in which a rib is elastically supported at two points. We will consider this case more closely with reference to our method of calculation. Of course the method is similar for a rib supported at only one point or at several points. It is assumed that the resultant of the two elastic supporting forces passes through the elastic axis of the wing when the structure is loaded in this axis, so that no new torsional moment is introduced. This arrangement is always structurally attainable and only simplifies the rather complex structural behavior.

Of course the other case can also be considered. Through the elastic support of two points of a rib, n new joint forces will be produced in the n joints of the rib. The resultant of these forces is calculated from the condition that the deflection of the unbraced wing, under the external load in the elastic axis of the wing, reduced by the easily calculable deflection of the unbraced wing under the sought resultant as the only load, must equal the actual depression of the supported rib due to the elastic support, which depression is proportional to the desired resultant. The distribution of the thus-determined resultant supporting force among the n rib joints is made according to proportion (2), wherewith, as before, the values of $K_{xy}^{(1)}$ are assignable. The values of $K_{xy}^{(2)}$ can be determined in a perfectly similar way. The magnitude of both elastic supporting forces, which must here form a couple, is determined from the condition that the twisting of the unbraced wing at the place of the elastic supporting forces under the external moment, reduced by the torsion of the same wing at the same point under the moment of the sought elastic supporting forces, must equal the actual torsion of the braced wing under the external moment, which can be determined from the elastic supporting condition. The distribution of this supporting moment among the individual rib joints is made according to proportion 3.

The superposing of $K_{xy}^{(1)}$ and $K_{xy}^{(2)}$ makes it possible to determine, for every loading condition, the actual joint forces K_{xy}, which can be improved, as previously, by consideration of the imperfect rigidity of the ribs. Since all statically indeterminate quantities are thus calculated, the moments of the spar and rib joints, the displacement of the joints and other important quantities can be determined in an elementary manner.

POLYGONAL SPAR PLAN

The great slenderness of the wing desired for aerodynamic reasons, on the one hand, and the tendency toward regular, unlimited spaces in the middle region of the wing structure, on the other hand, often influence designers to adopt the polygonal spar plan. We will explain briefly the application of our calculation method to this case according to Figure 5.

Otherwise the wing characteristics correspond to those of the general method. The calculation can be made in the simplest manner by first calculating all joint pressures as in the general case, i.e., without regard to the bend in the spar plan. Thereby we must sometimes only introduce the corresponding correct spar angle into the distribution proportions. According to the law of independence of rigid ribs, the joint forces in the central part of the wing are independent of the loads in the outer parts and vice versa, so that this procedure is permissible.

The effect of the bend in the spar is then expressed as if, at every joint of the rib l, there were in the plane of the rib a moment of the magnitude

$$D_{l y} \mp L_{l y} \sin (\bar{\alpha}_y - \alpha_y)/\sin \alpha_y,$$

where $L_{l y}$ are the spar moments at the joints l and y resulting from the external load. For the determination of the stresses and deformations of the rib l these supplementary torsional moments must be separately calculated at their points of application. For the calculation of the joint forces in the central part of the wing, which are produced by these supplementary torsional moments, the individual torsional moments are added algebraically, so that a supplementary torsional moment of the magnitude of the static moment of the external loads between l and m times cotan $\bar{\alpha}_s$ - cotan α_s acts on the rib as a whole. This supplementary torsional moment produces, in the central part of the wing, additional joint forces which can be easily calculated by the above-mentioned method and which are superposed on the previously determined joint forces. All the statically indeterminate quantities are thus known and both stresses and deformations can also be ascertained for this case.

DIRECTLY LOADED WING

The directly loaded wing has no fuselage for receiving the wing loads, all the loads carried by the airplane being distributed over the wing span, like the supporting forces of the air or of the ground. The more perfect the assimilation of these three force systems (loads, aerodynamic forces, ground forces) is from the outset, just so much less devolves on the spars for their equalization and just so much lighter will they, and consequently the structural weight of the airplane, be. The calculation of such wings is illustrated by Figure 6, which represents a very common type of wing in conformity with the requirements mentioned at the beginning. The loads are represented (of course, crudely), as is customary in other engineering structures, in the form of a uniform load per square meter. The sum of the given loads represents, in the former sense, the theoretical weight and differs, therefore, from the flying weight by the amount of the actual weight of the supporting structure, plating, etc., i.e., of the parts directly supported by the air without the intermediation of the supporting framework.

Simple loads (not safe ones) will be used as the basis of the calculation, because the investigation must apply to all flight and landing conditions, and the load factors will therefore differ. In our example we accordingly obtain loads of $a/6$ kg/m² or $b/6$ kg/m² under otherwise unchanged loading conditions. Corresponding to the sum of these reduced loads, we now choose the "influence loading" of a spar, whose distribution along the spar must accord with the distribution of the aerodynamic forces on the spar, whereby the spacing of the spars, a reduction at the wing tip, etc., must receive attention. The influence loading divided among the joints of any spar corresponding to the actual aerodynamic and landing forces must hold the theoretical loads, divided by the number of spars but otherwise unchanged, in perfect equilibrium as regards vertical displacement. The torsional equilibrium about the wing axis is obtained through a corresponding elevator moment acting on the middle rib, in accordance with the preliminary assumption.

All the joint forces are now determined for this loading condition. Then the influence load is allowed to pass to the next spar, the new requisite elevator moment is determined, and the desired quantities for this loading condition are also determined. This process is repeated as many times as there are spars. In such an investigation of all loading conditions, it is possible to plot, for every desired quantity, influence lines, whose ordinates represent the effect of the influence load, traveling from spar to spar, on the desired quantity at the desired point, and from which can then be determined each one of the stipulated flight and ground conditions without further calculation, but which also makes it possible to determine what effects follow from the interposition of the aerodynamic and ground forces which are not included in the stipulated loading conditions. Through systematic combination of the location of the external aerodynamic and landing forces and the corresponding load factors, it is possible to determine, for every member of the framework, the most unfavorable loading point, which can lie between the stipulated loading cases and to indicate their effect in the simplest manner, as is customary in regular engineering construction.

The calculation of the individual loading condition (one spar with the influence load, all other spars without load) differs in no way from the process described in the general part. This means, at first, displacement of the individual load series in the elastic axis, determination of the moment reaction at the middle rib, and calculation of the joint forces $K_{xy}^{(1)}$; then the calculation of the loading of the rigid ribs with the torsional moments produced by the displacement of the influence load in the static axis, finding the corresponding reaction moment in the middle rib, calculation of the joint forces $K_{xy}^{(2)}$, and lastly the algebraic addition of $K_{xy}^{(1)}$ and $K_{xy}^{(2)}$.

For the further calculations, determination of the bending moments, transverse forces, deformations, etc., each individual spar and rib can be separated from the wing structure and be treated as a single beam, which is stressed by the known theoretical and influence loads decisive for it and by the now-determined joint forces.

It must be especially emphasized that hitherto no concrete assumptions have been made concerning the course of the inertia moments along the spars and ribs and concerning their absolute magnitudes. These magnitudes can be determined only after the dimensional data are known. Moreover, the improvements, resulting from the difference between the actual and the assumed perfect rigidity of the ribs, similar to that indicated in the general part, can be calculated and considered.

Lastly, we will add something concerning the plotting and the practical interpretation of the influence lines obtained. Let us investigate spar 3 of the wing represented in Figure 6. From the calculated data for the four loading conditions of the whole wing we select the spar-joint moments $M_{x^3,y}$ of the given spar and enter it on a diagram of the wing as shown in Figure 7. The length of the ribs must be plotted according to scale, but the representation of the spar does not matter.

The influence lines must be interpreted according to the loading condition. Whether this is covered by one of the officially prescribed loading conditions A, B, C or D, cannot be foreseen, so that we will keep in mind the possibility that such may not be the case. If our further considerations are based on the characteristics of the Göttingen profile 449, and the calculation is made according to the D.V.L. loading assumptions for the strength calculation of airplanes, we obtain for our example the heavy lines in Figure 8 as the safe distribution of the aerodynamic forces along the wing chord for the standard loading cases A, B, C, and D. Since these aerodynamic forces are assumed as occurring in flight and represent "unfavorable" limiting cases, we are justified in assuming that every aerodynamic-force distribution between these limiting cases can occur, as several are plotted in fine lines between the principal loading cases in Figure 8. We have thereby coordinated the aerodynamic-force distribution interpolated between the principal loading cases (which therefore includes the pertinent maximum load factor) with the angles of attack of the wing.

If Figure 8 is drawn on tracing paper in the corresponding scale and laid on the influence lines of Figure 7, the aerodynamic-force distribution, decisive for the magnitude to be determined, can then be soon found, by a little testing and interpolating, from the condition that the arithmetic sum of the products of the aerodynamic-force ordinates times the influence-line ordinates must be a maximum. The evaluation of the decisive aerodynamic-force loading condition with the pertinent influence line is accomplished in the simplest manner by multiplying both areas, whereby the scale must be allowed for by a corresponding reduction. The landing forces can be represented in the same way as the aerodynamic forces by diagrams for the different landing conditions (bow landing, step landing, stern landing with all intermediate attitudes) and plotted exactly like the aerodynamic-force diagrams.

It is now known that all such calculations, especially the statically indeterminate systems, with their known degree of accuracy, allow statements concerning the inner force and stress distribution of the wing under operating stresses, but often do not suffice to draw conclusions regarding the actual structural safety, since decisive changes in the internal-force distribution generally occur through lack of conformity to Hook's law, especially above the permissible stresses before rupture.

The fact is reassuring that, according to the self-help principle of nature, these changes occur mostly in the direction of increasing the safety, as compared with that assumed from the elastic behavior, and that the materials now chiefly used in airplane construction show no pronounced yield points, which would particularly favor the occurrence of these force displacements, and, lastly, also the fact that fatigue of the wing components often results, in the elastic region, from exceeding the buckling or vibration strength, i.e., for a force distribution, such as was found in the elastic investigation.

SUMMARY

For multispar cantilever and semicantilever wings with parallel ribs under direct and indirect loading, a method is presented for approximate static calculation, which is based on the customary assumption of rigid ribs, while taking into account the systematic errors in the calculation results due to this arbitrary assumption. The procedure is given in greater detail for semicantilever and cantilever wings with polygonal spar plan form and for wings under direct loading only. The last example illustrates the advantages of the use of influence lines for such wing structures and their practical interpretation.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

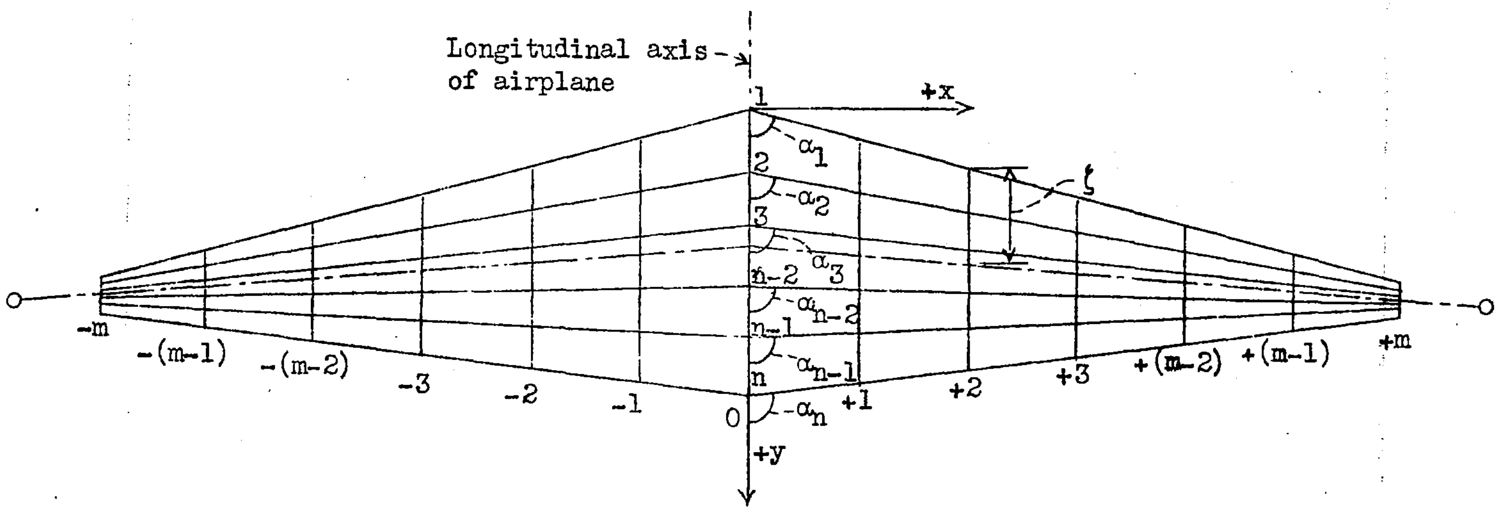

Fig. 1 Multispar wing structure with parallel ribs.

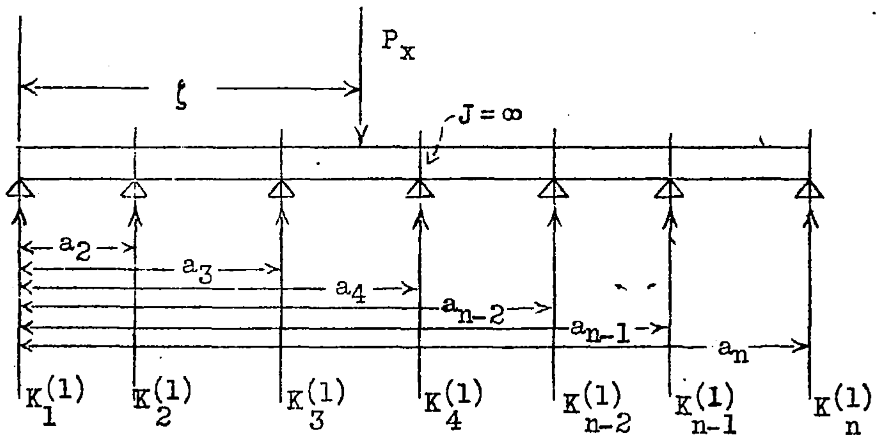

Fig. 2 Determination of elastic axis of wing.

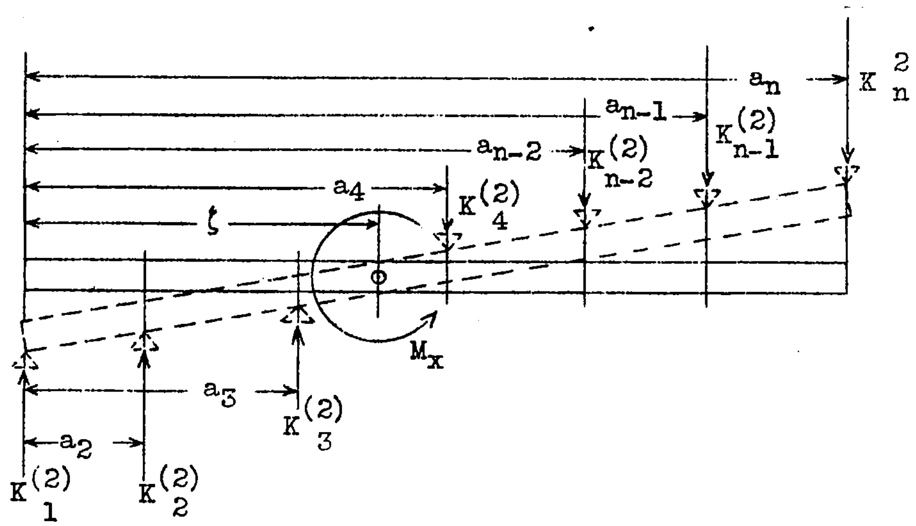

Fig.3 Torsional moment about elastic axis of wing.

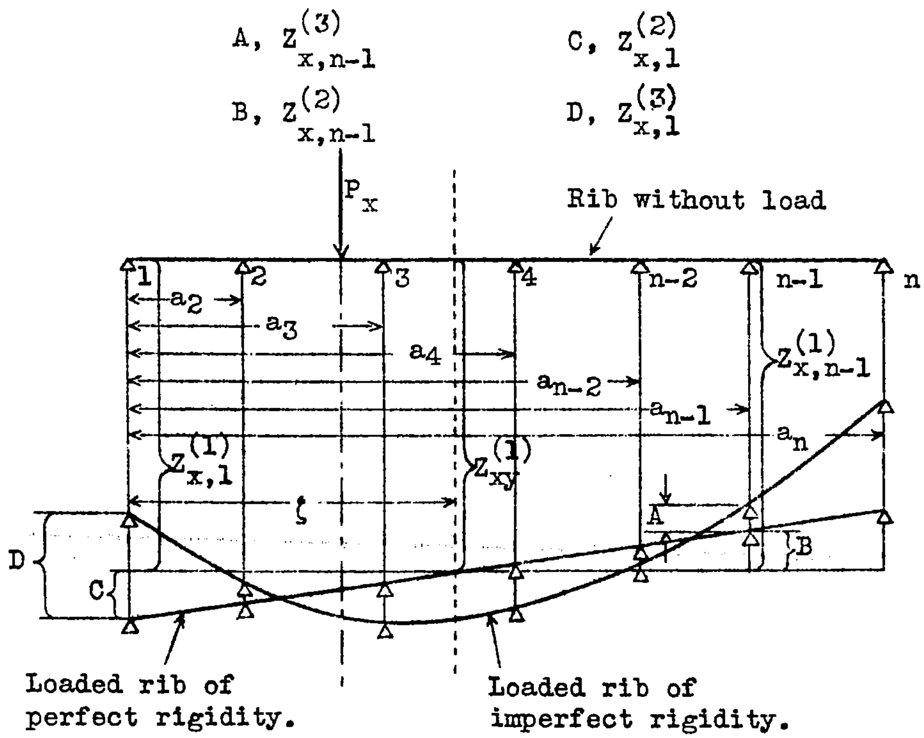

Fig.4 Representation of imperfect rigidity of rib.

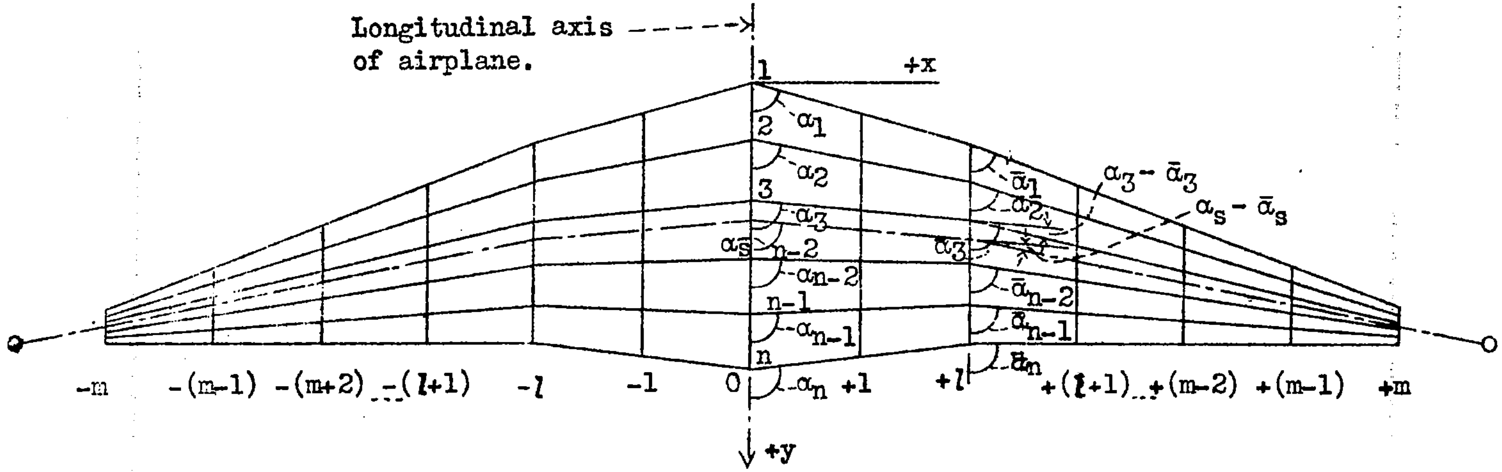

Fig. 5 Multispar wing structure with parallel ribs and polygonal spar plan form.

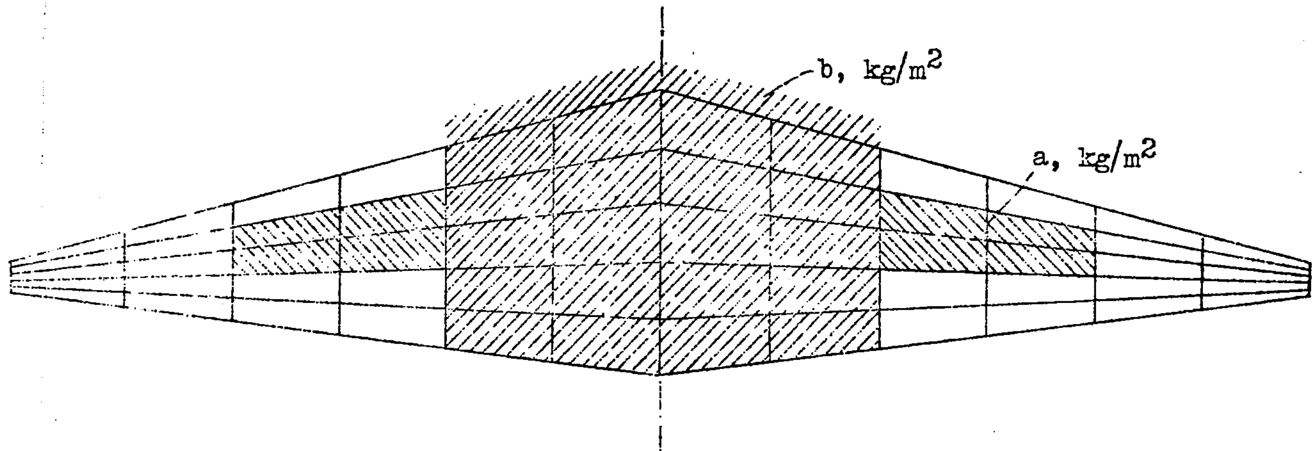

Fig. 6 Multispar wing structure with parallel ribs under direct loading.

$M_{xy,y}$ -------- Spar-joint moment at joints x,y produced by influence load on spar y.

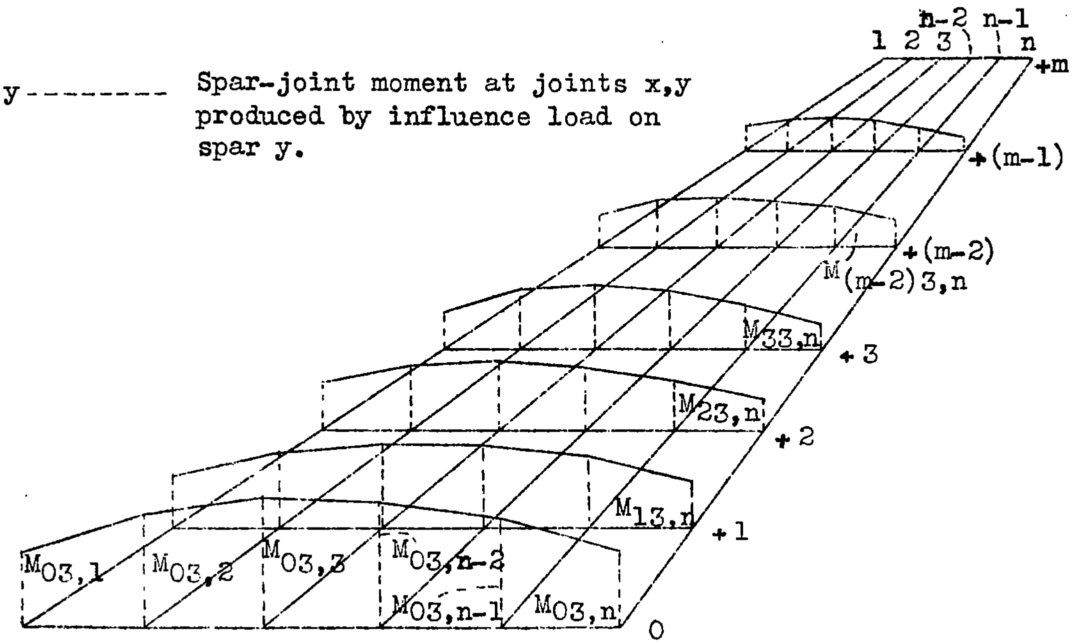

Fig. 7 Influence lines.

E , Wing chord
A , Case A and related
B , Case B and related
C , Case C and related
D , Case D and related

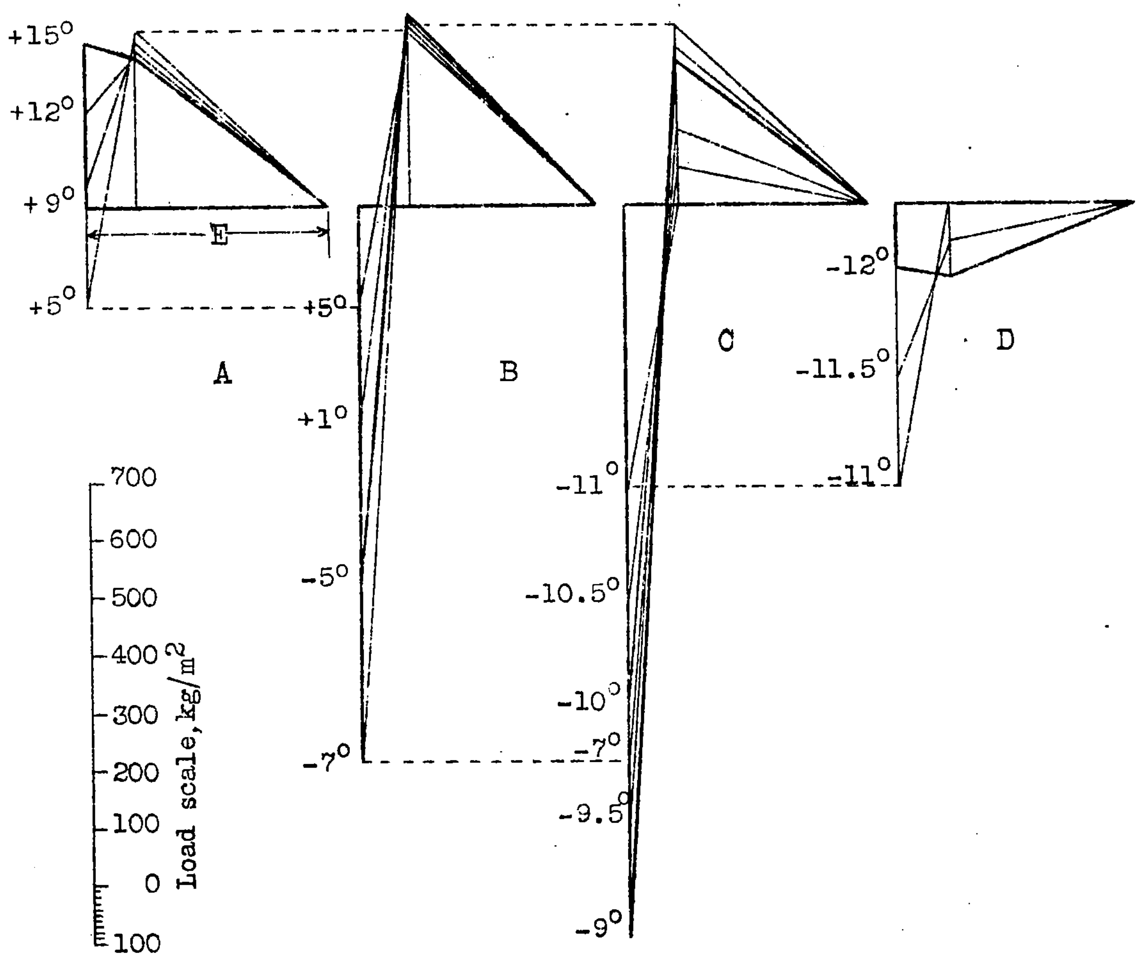

Fig.8 Principal and related loading cases.